Persistent Monitoring of Events with Stochastic Arrivals

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Supported by:
Ubiquity of Persistent Monitoring Tasks
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Arise whenever *spatially distributed events* must be continuously surveyed with *limited mobile resources*.
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Focus: Events with Stochastic Arrivals
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- *Transient* – the event soon expires/disappears afterwards
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- The **average rate of arrival** may be known roughly
Focus: Events with Stochastic Arrivals

Key characteristics:

- **Transient** – the event soon expires/disappears afterwards
- **Temporal stochasticity** – arrival time is unknown *a priori*
- The **average rate of arrival** may be known roughly
- Collecting such events requires *waiting* at the event sites
Problem Formulation
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\[ \lambda_1 = 0.5 \]

\[ \lambda_2 = 1.3 \]

\[ \lambda_3 = 2.5 \]

\[ \lambda_4 = 1.2 \]

\[ \lambda_5 = 1.6 \]

\[ \lambda_6 = 0.9 \]
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- Determining a policy $\pi = (t_1, \ldots, t_n)$ that
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- Determining a policy $\pi = (t_1, ..., t_n)$ that
- Maximizes the amount of events observed in a balanced way
- Minimizes the maximum delay between event observations
Problem Formulation, cont.

- A mobile sensor patrols a cyclic route along sites $S = \{s_1, ..., s_n\}$
- Events at site $s_i$ follow a Poisson process of intensity $\lambda_i$
- Travelling from site $i$ to site $j$ takes $\tau_{ij}$ time; total travel time $T_{tr} := \sum \tau_{ij}$.
- Also, given policy $\pi = (t_1, ..., t_n)$, let observation time $T_{obs} := \sum t_i$, cyclic period $T := T_{obs} + T_{tr}$

Find policy $\pi^* = (t_1^*, ..., t_n^*)$, over an infinite horizon
- Maximize data collecting effort in a balanced manner, measured by ($N_i(\pi)$ is the total events observed at $s_i$)

$$J_1(\pi) = \min_i \frac{\mathbb{E}[N_i(\pi)]}{\sum_{j=1}^{n} \mathbb{E}[N_j(\pi)]}$$

- Minimize maximum data collection delay ($T_i(\pi)$) between consecutive visits to the same site

$$J_2(\pi) = \max_i \mathbb{E}[T_i(\pi)]$$
Related Work

**Persistent monitoring**: Michael et al. (2011); Smith et al. (2011); Alamdari et al. (2012); Arvelo et al. (2012); Cassandras et al. (2013); Girard et al. (2004); Grocholsky et al. (2006); Lan and Schwager (2013); Nigam and Kroo (2008); Smith et al. (2012); Soltero et al. (2012).

**Sensor scheduling**: Fuemmeler and Veeravalli (2008); He and Chong (2004); Hero et al. (2008); Ny et al. (2008).

**Coverage**: Choset (2000, 2001); Gabriely and Rimon (2003); Chin and Ntafos (1988); Hokayem et al. (2008); Ntafos (1991).
Main Result

- There is an *uncountably infinite* set of policies, $\Pi$, that maximizes $J_1(\pi)$
- There is a *unique policy* $\pi^* = (t_1^*, \ldots, t_n^*) \in \Pi$ that minimizes $J_2(\pi)$
- This is due to the *quasi-convexity* of $\mathbb{E}[T_i(\pi)]$ ($J_2(\pi) = \max_i \mathbb{E}[T_i(\pi)]$)

- Furthermore, we can *efficiently* compute $\pi^*$: $O(n)$ time for $n$ sites, and then $O(\log n)$ for adding/removing sites
Optimizing the First Objective $J_1(\pi)$

Recall that $\pi := (t_1, \ldots, t_n)$, $N_i(\pi) :=$ total number of events observed at site $s_i$
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$$J_1(\pi) = \min_i \frac{\mathbb{E}[N_i(\pi)]}{\sum_{j=1}^n \mathbb{E}[N_j(\pi)]} = \min_i \frac{\lambda_i t_i}{\sum_{j=1}^n \lambda_j t_j}$$
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$$\max_{\pi} J_1(\pi) \Rightarrow \lambda_1 t_1 = \cdots = \lambda_n t_n$$
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$$\sum_{i=1}^n t_i = T_{obs}$$
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$$\sum_{i=1}^{n} t_i = T_{obs} \quad \Rightarrow \quad t_i = \sigma \frac{T_{obs}}{\lambda_i} = \frac{T_{obs}}{\lambda_i \sum_{j=1}^{n} \frac{1}{\lambda_j}}$$
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$T_{obs}$ is arbitrary

$\Rightarrow$ *uncountably infinite* number of policies that maximizes $J_1(\pi)$
Computing the Expected Delay $\mathbb{E}[T_i(\pi)]$
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\[ T = T_{obs} + T_{tr} \]
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$T_i(\pi)$
Computing the Expected Delay $\mathbb{E}[T_i(\pi)]$

$$
T_i = T_{\text{obs}} + T_{\text{tr}}
$$
Computing the Expected Delay $\mathbb{E}[T_i(\pi)]$

\[ E_m := \mathbb{E}[t_{left} + t_{right} + T - t_i + mT] \]
Computing the Expected Delay $\mathbb{E}[T_i(\pi)]$

\[ \mathbb{E}_m := \mathbb{E}[t_{left} + t_{right} + T - t_i + mT] = 2\mathbb{E}[t_{left}] + (m + 1)T - t_i \]
Computing the Expected Delay $\mathbb{E}[T_i(\pi)]$

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$$
= 2 \left( \frac{1}{\lambda_i} - \frac{t_i e^{-\lambda_i t_i}}{1 - e^{-\lambda_i t_i}} \right) + (m + 1)T - t_i
$$
Computing the Expected Delay $\mathbb{E}[T_i(\pi)]$

\[ T = T_{obs} + T_{tr} \]

\[ t_i \]

\[ t_{left} \quad T - t_i \quad mT \quad t_{right} \]

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\[ \mathbb{E}[T_i(\pi)] = p_m \mathbb{E}_m = e^{-m\lambda_i t_i} (1 - e^{-\lambda_i t_i}) \mathbb{E}_m \]
Computing the Expected Delay $\mathbb{E}[T_i(\pi)]$

\[ T = T_{\text{obs}} + T_{\text{tr}} \]

\[ t_i \quad t_{\text{left}} \quad T - t_i \quad mT \quad t_{\text{right}} \quad t \]

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Computing the Expected Delay $\mathbb{E}[T_i(\pi)]$

$$
\begin{align*}
\mathbb{E}_m & := \mathbb{E}[t_{left} + t_{right} + T - t_i + mT] = 2\mathbb{E}[t_{left}] + (m + 1)T - t_i \\
& = 2 \left( \frac{1}{\lambda_i} - \frac{t_i e^{-\lambda_i t_i}}{1 - e^{-\lambda_i t_i}} \right) + (m + 1)T - t_i \\
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t_i & = \frac{T_{obs}}{\lambda_i \sum_{j=1}^{n} \frac{1}{\lambda_j}} = \frac{2}{\lambda_i} - \frac{T_{obs} + T_{tr} + \gamma_i T_{obs} - \gamma_i T_{obs} e^{-\lambda_i \gamma_i T_{obs}}}{1 - e^{-\lambda_i \gamma_i T_{obs}}} \\
\gamma_i & := \frac{1}{\lambda_i \sum_{j=1}^{n} \frac{1}{\lambda_j}}
\end{align*}
$$
$\mathbb{E}[T_i(\pi)]$ as Function of $T = T_{obs} + T_{tr}$
\[ \mathbb{E}[T_i(\pi)] \text{ as Function of } T = T_{obs} + T_{tr} \]

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<tr>
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<tr>
<td>( \lambda' ) (1/hr)</td>
<td>0.5</td>
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![Graph showing expected delay (hrs) vs $T$ (policy period hrs)]
$\mathbb{E}[T_i(\pi)]$ as Function of $T = T_{obs} + T_{tr}$, cont.
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\[ \mathbb{E}[T_i(\pi)] \] as Function of \( T = T_{obs} + T_{tr} \), cont.
\( \mathbb{E}[T_i(\pi)] \) as Function of \( T = T_{obs} + T_{tr} \), cont.

Key properties of \( \mathbb{E}[T_i(\pi)] \)

- **Monotonic** in \( \lambda_i \)
\( \mathbb{E}[T_i(\pi)] \) as Function of \( T = T_{obs} + T_{tr} \), cont.

Key properties of \( \mathbb{E}[T_i(\pi)] \):
- **Monotonic** in \( \lambda_i \)
- Appears to be **convex**
$\mathbb{E}[T_i(\pi)]$ as Function of $T = T_{obs} + T_{tr}$, cont.

Key properties of $\mathbb{E}[T_i(\pi)]$

- **Monotonic** in $\lambda_i$
- Appears to be **convex** ← actually, **quasi-convex**
Minimizing $J_2(\pi)$ and the Algorithmic Perspective

$$\{\lambda_i\} \Rightarrow \gamma_i := \frac{1}{\lambda_i \sum_{j=1}^{n} \frac{1}{\lambda_j}}$$
Minimizing $J_2(\pi)$ and the Algorithmic Perspective

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\[ \{\tau_{ij}\} \Rightarrow T_{tr} \]
Minimizing $J_2(\pi)$ and the Algorithmic Perspective

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\[ \{\tau_{ij}\} \Rightarrow T_{tr} \]

\[ \Rightarrow \mathbb{E}[T_i(\pi)] = \frac{2}{\lambda_i} - \frac{T_{obs} + T_{tr} + \gamma_i T_{obs} - \gamma_i T_{obs}e^{-\lambda_i \gamma_i T_{obs}}}{1 - e^{-\lambda_i \gamma_i T_{obs}}} \]
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![Graph showing the relationship between $\lambda$ and $\mathbb{E}[T_i(\pi)]$ for different values of $\lambda$.]
Minimizing $J_2(\pi)$ and the Algorithmic Perspective

$$\{\lambda_i\} \Rightarrow \gamma_i := \frac{1}{\lambda_i \sum_{j=1}^{n} \frac{1}{\lambda_j}}$$

$$\{\tau_{ij}\} \Rightarrow T_{tr}$$

$$\Rightarrow \mathbb{E}[T_i(\pi)] = \frac{2}{\lambda_i} - \frac{T_{obs} + T_{tr} + \gamma_i T_{obs} - \gamma_i T_{obs} e^{-\lambda_i \gamma_i T_{obs}}}{1 - e^{-\lambda_i \gamma_i T_{obs}}}$$

$$\implies \max_i \mathbb{E}[T_i(\pi)]$$

![Graph showing the relationship between $\lambda$ and $\mathbb{E}[T_i(\pi)]$ for different values of $\lambda$. The graph compares $\lambda$ values of 0.5, 1.3, 2.5, 1.2, 1.6, and 0.9. The curve for $\lambda = 0.5$ is the highest, indicating a decrease in $\mathbb{E}[T_i(\pi)]$ as $\lambda$ increases.]
Minimizing $J_2(\pi)$ and the Algorithmic Perspective

$\{\lambda_i\} \Rightarrow \gamma_i := \frac{1}{\lambda_i \sum_{j=1}^{n} \frac{1}{\lambda_j}} \left\{ \{t_{ij}\} \Rightarrow T_{tr} \right\} \Rightarrow \mathbb{E}[T_i(\pi)] = \frac{2}{\lambda_i} - \frac{T_{obs} + T_{tr} + \gamma_i T_{obs} - \gamma_i T_{obs} e^{-\lambda_i \gamma_i T_{obs}}}{1 - e^{-\lambda_i \gamma_i T_{obs}}}$

\[\max_{i} \mathbb{E}[T_i(\pi)]\]
\[\min_{\pi} \max_{i} \mathbb{E}[T_i(\pi)]\]
Minimizing $J_2(\pi)$ and the Algorithmic Perspective

\[
\{\lambda_i\} \Rightarrow \gamma_i := \frac{1}{\lambda_i \sum_{j=1}^{n} \frac{1}{\lambda_j}} \quad \left\{ \begin{array}{l}
\{\tau_{ij}\} \Rightarrow T_{tr} \\
\end{array} \right. \Rightarrow \mathbb{E}[T_i(\pi)] = \frac{2}{\lambda_i} - \frac{T_{obs} + T_{tr} + \gamma_i T_{obs} - \gamma_i T_{obs} e^{-\lambda_i \gamma_i T_{obs}}}{1 - e^{-\lambda_i \gamma_i T_{obs}}}
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\[T^* = T_{obs} + T_{tr}\]
Minimizing $J_2(\pi)$ and the Algorithmic Perspective

\begin{align*}
\{\lambda_i\} & \Rightarrow \gamma_i := \frac{1}{\lambda_i \sum_{j=1}^{n} \frac{1}{\lambda_j}} \\
\{\tau_{ij}\} & \Rightarrow T_{tr} \\
& \Rightarrow \mathbb{E}[T_i(\pi)] = \frac{2}{\lambda_i} - \frac{T_{obs} + T_{tr} + \gamma_i T_{obs} - \gamma_i T_{obs} e^{-\lambda_i \gamma_i T_{obs}}}{1 - e^{-\lambda_i \gamma_i T_{obs}}}
\end{align*}

\[ t_i = \gamma_i T_{obs} \]

$T^* = T^*_{obs} + T_{tr} \Rightarrow \pi^* = (t^*_1, \ldots, t^*_n)$
Minimizing $J_2(\pi)$ and the Algorithmic Perspective

\[
\{\lambda_i\} \Rightarrow y_i := \frac{1}{\lambda_i \sum_{j=1}^{n} \frac{1}{\lambda_j}} \quad \Rightarrow \quad \mathbb{E}[T_i(\pi)] = \frac{2}{\lambda_i} - \frac{T_{obs} + T_{tr} + y_i T_{obs} - y_i T_{obs} e^{-\lambda_i y_i T_{obs}}}{1 - e^{-\lambda_i y_i T_{obs}}}
\]

\[
\{\tau_{ij}\} \Rightarrow T_{tr}
\]

\[
\begin{align*}
\text{max } \mathbb{E}[T_i(\pi)] & \quad \Rightarrow \quad \max_i \mathbb{E}[T_i(\pi)] \\
\text{min } \max_{\pi} \mathbb{E}[T_i(\pi)] & \quad \Rightarrow \quad \min_{\pi} \max_i \mathbb{E}[T_i(\pi)]
\end{align*}
\]

- Computing $\{y_i\}$ takes $O(n)$ time

\[
T^* = T_{obs}^* + T_{tr} \quad \Rightarrow \pi^* = (t_1^*, \ldots, t_n^*)
\]
Minimizing $J_2(\pi)$ and the Algorithmic Perspective

\[
\{\lambda_i\} \Rightarrow \gamma_i := \frac{1}{\lambda_i \sum_{j=1}^{n} \frac{1}{\lambda_j}} \quad \Rightarrow \\
\{\tau_{ij}\} \Rightarrow T_{tr} \\
\Rightarrow \mathbb{E}[T_i(\pi)] = \frac{2}{\lambda_i} - \frac{T_{obs} + T_{tr} + \gamma_i T_{obs} - \gamma_i T_{obs}e^{-\lambda_i \gamma_i T_{obs}}}{1 - e^{-\lambda_i \gamma_i T_{obs}}} \\
\]

- Computing $\{\gamma_i\}$ takes $O(n)$ time
- Rest operation takes $O(1)$ time

\[T^* = T_{obs}^* + T_{tr} \Rightarrow \pi^* = (t_1^*, ..., t_n^*)\]
Minimizing $J_2(\pi)$ and the Algorithmic Perspective

\[
\begin{aligned}
\{\lambda_i\} \Rightarrow \gamma_i & := \frac{1}{\lambda_i \sum_{j=1}^{n} \frac{1}{\lambda_j}} \\
\{\tau_{ij}\} \Rightarrow T_{tr} & \Rightarrow \mathbb{E}[T_i(\pi)] = \frac{2}{\lambda_i} - \frac{T_{obs} + T_{tr} + \gamma_i T_{obs} - \gamma_i T_{obs} e^{-\lambda_i \gamma_i T_{obs}}}{1 - e^{-\lambda_i \gamma_i T_{obs}}} \\
\end{aligned}
\]

- Computing $\{\gamma_i\}$ takes $O(n)$ time
- Rest operation takes $O(1)$ time
- Incremental computation takes $O(\log n)$ time

\[T^* = T_{obs}^* + T_{tr} \Rightarrow \pi^* = (t_1^*, \ldots, t_n^*)\]
Conclusion and Future Work

Summary of contribution
- Introduced a persistent monitoring problem of events with stochastic arrival
- Fully characterized a multi-objective optimization problem to allow efficient computation of the optimal cyclic patrolling policy
- Convergence rate, robustness, global uniqueness of solution

Future research
- Feedback-based policies (onboard data processing v.s. off-line)
- General traveling topology (in addition to cycle graphs)
- Information theoretic approach